Nonlinear Global and Local Document Degradation Models

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ABSTRACT

Two sources of document degradation are modeled: 1) perspective distortion that occurs while photocopying or scanning thick, bound documents; and 2) degradation due to perturbation in the optical scanning and digitization process: speckle, blurr, jitter, and threshold. Perspective distortion is modeled by studying the underlying perspective geometry of the optical system of photocopiers and scanners. An illumination model is described to account for the nonlinear intensity change occurring across a page in a perspective-distorted document. The optical distortion process is modeled morphologically. First, a distance transform on the foreground is performed; this is followed by a random inversion of binary pixels in which the probability of flip is a function of the distance of the pixel to the boundary of the foreground. Correlating the flipped pixels is modeled by a morphological closing operation. © 1995 John Wiley & Sons, Inc.

I. INTRODUCTION

There are many reasons for modeling document degradation. First, to study the performance of any OCR algorithm, it is necessary to characterize the perturbation in the output performance as a function of the perturbation in the input [1–4]. This is possible only if we have a perturbation/degradation model for the input document. Second, a degradation model permits the evaluation of an algorithm for a continuum of degradation levels, from low to high degradation levels. This, in turn, allows us to locate the "break-down" point or the "knee" of the algorithm, which is not available in the commonly used evaluation methods, such as confusion matrices. Third, a knowledge of the degradation model can enable us to design algorithms for restoring degraded documents. Furthermore, OCR algorithm designers can make use of these degradation models explicitly rather than implicitly, as is usually done in current literature.

In this article we model two sources of document degradation: 1) perspective distortion that occurs while photocopying or scanning thick, bound documents; and 2) degradation due to perturbation in the optical process; speckle, blur, jitter, threshold, and so forth. Perspective distortion is modeled by studying the underlying perspective geometry of the optical system of photocopiers and scanners. An illumination model is proposed to account for the nonlinear intensity change occurring across a page in a perspective-distorted document. The local optical distortion process is modeled morphologically. First, a distance transform on the foreground is performed; this is followed by a random inversion of

binary pixels, in which the probability of flip is a function of the distance of the pixel to the boundary of the foreground. Correlating the flipped pixels is modeled by a morphologic closing operation. A less detailed version of this article was published in International Conference on Document Analysis and Recognition [5].

Baird [6] discusses a model for character degradation. His model does not account for the nonlinear distortions produced from perspective distortions. In [7], Baird discusses the various uses of document degradation models. Loce [8] models the perturbation introduced as a result of mechanical disturbances in high-end Xerox photocopiers. Our article models the distortions in geometry and illumination due to perspective. This is a page level distortion model, as opposed to the noise model proposed by Baird, which works at character and pixel level. Furthermore, our model of the optical process is morphological and more conducive for degradation parameter estimation. Maltz [9] has done a transfer function analysis of the xerography process.

II. OPTICAL SETUP

A typical setup for scanners and photocopiers is shown in Figure 1, a book to be photocopied. The page to be photocopied is not flat on the document glass because the book is tightly bound and the spine of the book is thick. We model four sources of degradation in the following sections.
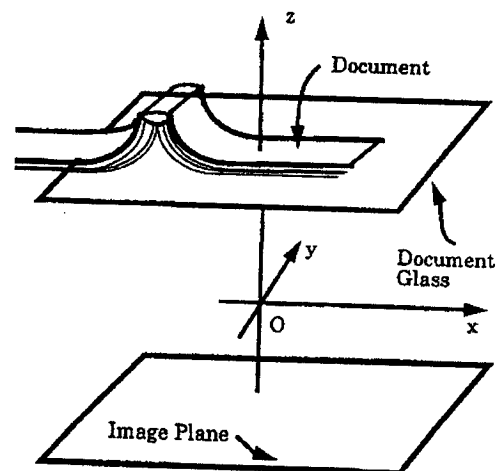
Please see [16] and [17] for related discussions.

Figure 1. The setup while photocopying a thick, bound document. The center of perspectivity is at O, which is also the origin of the coordinate frame.

III. DEFORMATION MODEL FOR THE PHYSICAL PAGE BENDING PROCESS

First, the page itself undergoes a physical deformation, in which the document page goes through a bending process near the spine of a thick, bound document. The page is no longer a flat surface on the document glass but a curved surface bending away from the glass near the spine of the book. We model this curved portion of the document page as a circular arc segment along the x axis and assume that there is no such deformation along the y axis. The global rotation and translation can be modeled in another stage. Figure 2 illustrates this deformation phenomenon.

Let $A = (x_a, y_a, f)'$, $B = (x_b, y_b, f)'$. Furthermore, let ρ be the radius of the deformation circle and let the bent segment subtend an angle θ at the center of the circle D. Let the point A map to the point $A' = (x_{a'}, y_{a'}, z_{a'})'$ after deformation. Then the coordinates of A' are given by

$$x_{a'} = x_a + \rho(\theta - \sin \theta) \tag{1}$$

$$y_{a'} = y_a \tag{2}$$

$$z_{a'} = f + \rho(1 - \cos \theta) \tag{3}$$

Let the point $P = (x_p, y_p, f)'$ be such that $x_a \leq x_p \leq x_b$, and let P map to the point $P' = (x_{p'}, y_{p'}, z_{p'})'$ after deformation. Let the angle subtended by the arc $P'C$ at the center D be ϕ, where

$$\phi = (x_a + \rho\theta - x_p)/\rho = \theta - (x_p - x_a)/\rho . \tag{4}$$

Now the coordinates of P' can be calculated as given below:

$$x_{p'} = x_p + \rho(\phi - \sin \phi) \tag{5}$$

$$y_{p'} = y_p \tag{6}$$

$$z_{p'} = f + \rho(1 - \cos \phi) . \tag{7}$$

Note that for points P in the original document with $x_p > x_b$, we have no deformation, and hence $P' = P$.

IV. PERSPECTIVE DISTORTION MODEL

The bending deformation is followed by a perspective distortion where the point P' on the document maps to the point P'' on the image (Figure 3). Let the focal length of the optical system be f, and let the center of perspectivity, O, be at the origin. Assume that the image plane is at the focal plane at $-f$. Let $P'' = (x_{p''}, y_{p''}, z_{p''})'$ be the perspective projection of the point P' on the document page. The coordinates of P'' are given by the following equations [1, 10]:

$$x_{p''} = -f(x_{p'}/(f + \rho(1 - \cos \phi))) \tag{8}$$

$$= -f((x_p + \rho(\phi - \sin \phi))/(f + \rho(1 - \cos \phi))) \tag{9}$$

$$y_{p''} = -f(y_{p'}/(f + \rho(1 - \cos \phi))) \tag{10}$$

$$= -f(y_p/(f + \rho(1 - \cos \phi))) \tag{11}$$

$$z_{p''} = -f . \tag{12}$$

Note that for points P in the original document with $x_p > x_b$, we have no bending or perspective deformation and hence $-P'' = P' = P$.

V. NONLINEAR ILLUMINATION MODEL

Because the document page is no longer flat, but a curved surface, the illumination on the document is not constant. The illumination at a point P' on the document pages is inversely
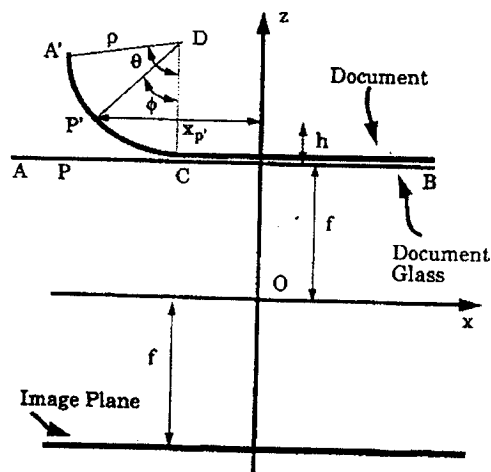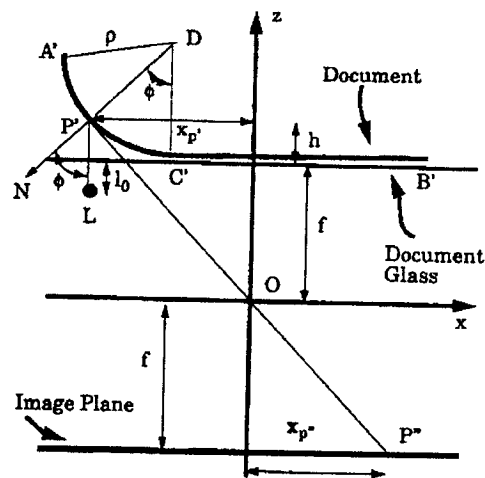
Figure 2. The bending deformation of the document pages. The side view of Figure 1 while looking in the positive y direction is shown. The points A', B', and C' on the document page would have been at the points A, B, and C on the document glass if the page had not been curved. The curve $A'P'C$ is modeled as a circular arc segment that subtends an angle θ at the center, D, of the circle, which has a radius ρ. Here $h = \rho(1 - \cos \theta)$ and $x_{p'} = x_p + \rho(\phi - \sin \phi)$, where $\phi = \theta - (x_p - x_a)/\rho$. Rest of the page from C to B along the x axis is not deformed. It is assumed that the page does not undergo any deformation in the y direction, either.

Figure 3. Perspective distortion. The point P' on the bent document page projects to the point P'' on the image plane. The coordinates of P'' are given as $x_{p''} = -f \cdot x_{p'}/(f + h)$, $y_{p''} = -f \cdot y_{p'}/(f + h)$, and $z_{p''} = -f$, where $h = \rho(1 - \cos \phi)$ and $\phi = \theta - (x_p - x_a)/\rho$.

proportional to the distance of point P' from the light source L. The light source L moves below the document glass from one end to the other. Let the distance between the document glass, and the light source L be l_0. See figure 3. At the places where the page is curved the distance between the light source and the document pages is $l = l_0 + \rho(1 - \cos \phi)$ where ϕ is the angle arc, $P'B$ subtends at B. Note ϕ is also the angle between the normal at P' and the negative z direction. We model the illumination as a diffuse lighting model. Thus the intensity of light is proportional to the cosine of the angle ϕ. Furthermore, after reflection, the diffuse model assumes the intensity of light is same in all directions [1, 10]. Let I_0 be the intensity at a point where the document is not curved—that is, the distance between the light and the point under consideration is l_0. Thus,

$$I_0 \propto 1/l_0^2 . \tag{13}$$

Next, the intensity at I_p, a point on the curved part is proportional to $\cos \phi$ and inversely proportional to $(l_0 + \rho(1 - \cos \phi))^2$. Thus,

$$I_p \propto \cos \phi / [l_0 + \rho(1 - \cos \phi)]^2 . \tag{14}$$

Thus, taking a ratio of these two equations, we have

$$I_p = I_0(l_0/(l_0 + \rho(1 - \cos \phi)))^2 . \tag{15}$$

Under the assumption of diffuse lighting, we have $I_{p'} = I_{p''}$.

VI. NONLINEAR OPTICAL POINT-SPREAD FUNCTION

In an imaging process if a point P' is not in the focal plane, it is not in focus in the image plane if the image plane is at f. In fact, the image of a point geometrically is a disk if the image plane is not in focus [1, 10–12] (Figure 4). If Δ is the diameter of the lens, and h is the distance of the image plane from the focal plane, then the diameter of the disk is given by

$$d = \Delta(h/f) . \tag{16}$$

But because of optical irregularities, in reality we do not get a disk as the image but blurred version of a disk. In fact, this blurred disk can be modeled as a Gaussian with a standard deviation $\sigma = k \cdot d$, where k is a camera constant.
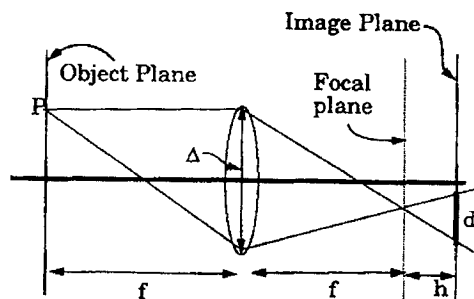
Notice that in our case, the distance of a point on the document page is in focus only if the document page is on the document glass (the focal plane). The curved region in particular is not in focus, because the points in that region are different distances from the focal plane. Thus, the amount of blurring that a point goes through is different for the points on the curved segment.

Algorithmically, after performing the bending transformation, perspective distortion, and nonlinear illumination, another stage is necessary, in which the image is convolved with a space-varying Gaussian kernel. The kernel has a standard deviation σ given by

$$\sigma = k \cdot \rho(1 - \cos \phi) \tag{17}$$

in the curved regions and constant σ_0 elsewhere.

VII. SIMULATION OF THE PERSPECTIVE DISTORTION MODEL

In this section we show some simulation results of the model discussed thus far. The original non-distorted image is shown in Figure 5. The dimensions of the image are 201×201. The convolution kernel size used was 5×5. Two perspective deformations are shown in Figures 6 and 7. The parameters, in units of pixels, used for generating Figure 6 were:

$$\rho = 152.87 \tag{18}$$

$$\theta = 30° \tag{19}$$

$$f = 80 \tag{20}$$

$$\Delta = 20 \tag{21}$$

$$k = 8 \tag{22}$$

$$l_0 = 10 \tag{23}$$

Figure 4. If the image plane is not at focus, then a point P maps to a disk of radius d. If the diameter of the lens is Δ and the focal length is f, the disk has a diameter $d = \Delta \cdot (h/f)$. Note that in the real world the disk becomes blurred and can be approximated by a Gaussian. See text for more details.
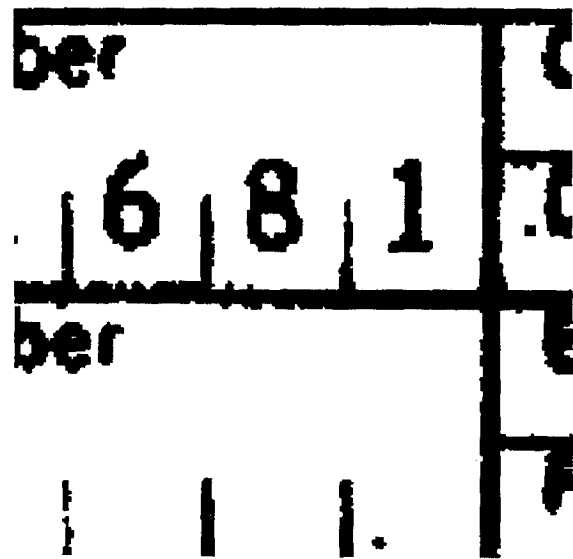
Figure 5. The original binary image before undergoing perspective distortion.
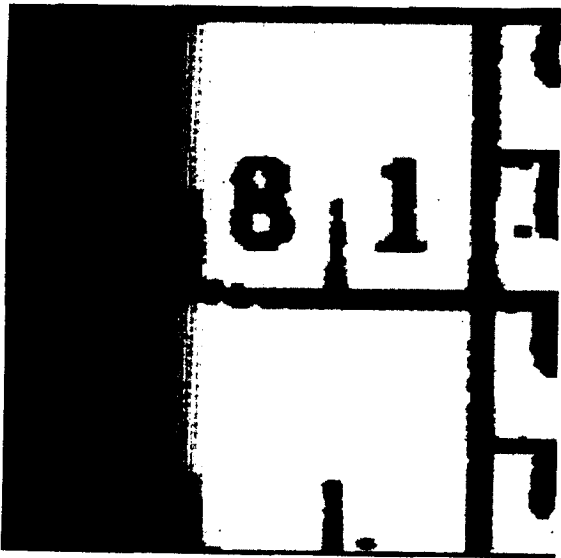
Figure 6. This image is produced after undergoing perspective distortion. Notice that the bend is very gradual, and the intensity of light decreases as you go along the curved region. Furthermore, the text is no longer horizontal but curved inward. In addition, the blurring gets progressively worse toward the left edge of the image.

The parameters used for generating Figure 7 were:

$$\rho = 95.54 \qquad (24)$$
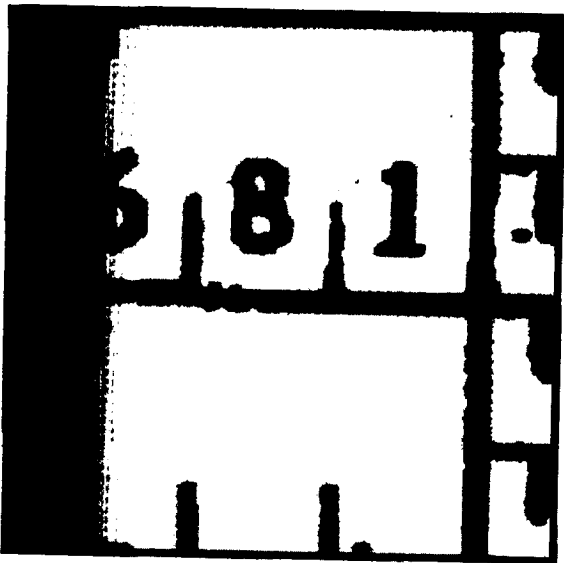
$$\theta = 30° \qquad (25)$$

Figure 7. This image is produced after undergoing perspective distortion that is sharper than the previous image. Also, the intensity variation is not as pronounced as the previous image.

$$f = 50 \qquad (26)$$

$$\Delta = 50 \qquad (27)$$

$$k = 1 \qquad (28)$$

$$l_0 = 20 \qquad (29)$$

VIII. MORPHOLOGIC MODEL FOR LOCAL DISTORTION

In this section we discuss a local document degradation model. This noise model is based on the distance transform [1, 13] of the ground truth data and some morphological postprocessing. We model the probability of a pixel changing from its ideal value as a function of the distance of that pixel from the boundary of a character. Let d be the distance (four connected or eight connected) of a foreground or background pixel from the boundary of the character and α and β be scale parameters. Let $P(1|d, \beta, f)$ and $P(0|d, \beta, f)$ be the probability of a foreground pixel at a distance d to remain as 1 and to change to a 0, respectively. Similarly, let $P(1|d, \alpha, b)$ and $P(0|d, \alpha, b)$ be the probability of a background pixel at a distance d changing to a 1 and remaining a 0, respectively. The functions $P(1|d, \alpha, f)$ and $P(1|d, \alpha, b)$ could be different. The random perturbation process then proceeds to change pixel values in a pixel by a pixel-independent manner. This is followed by a morphological closing operation to account for the correlation introduced by the optical point-spread function preceding the thresholding operation that produces the noisy image.

The following forms for the background and foreground conditional probabilities were used in the images shown in Figure 8.

$$P(1|d, \alpha, b) = 1 - P(0|d, \alpha, b) = \alpha_0 e^{-\alpha d^2} + \eta \qquad (30)$$

$$P(0|d, \beta, f) = 1 - P(1|d, \beta, f) = \beta_0 e^{-\beta d^2} + \eta \qquad (31)$$

The closing operation was performed with a 2×2 binary structuring element.

The document degradation model proposed by Baird [6, 14] models the physical degradation process. The degradation model is parameterized; some of the parameters are blur, speckle, jitter, threshold values, size, and rotation. Three major noise parameters are blur, speckle, and the threshold value. In Figure 9 we show the simulation of the model for fixed blur, speckle, and threshold values.

The original noise-free image of the document page is shown in Figure 10. Figures 11–14 gives examples of artificially degraded full-page documents using the local document degradation model described in this section. The images are 3300×2550 and correspond to a page size of 11×8.5 in. sampled at 300 samples/in. The parameter values used for Figures 11 and 12 result in a "thinning" degradation, in which the characters tend to become thinner and may break into multiple pieces. The parameters values used for Figures 13 and 14 result in a "blurring effect," in which the characters tend to become thicker.
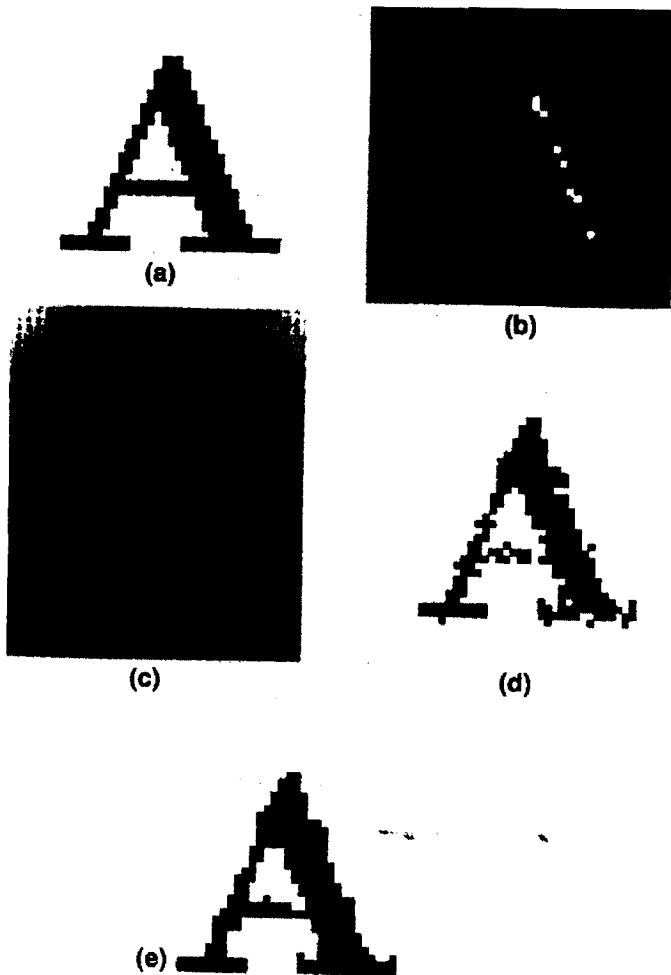
(a)

(b)

(c)

(d)

(e)

Figure 8. Morphologic local distortion model: (a) ground truth data; (b) distance transforms of foreground (a); (c) distance transforms of background (a); (d) the resultant perturbed image with exponential probability distribution $P(0|d, \beta, f) = P(1|d, \alpha, b) = \alpha_0 e^{-\alpha d^2}$ and $\alpha = \beta = 2$, $\alpha_0 = \beta_0 = 1$; (e) closing of result in (d) by a 2×2 binary structuring element.

Because the input to the degradation software can be any LaTex-formatted ASCII file, the same text can be formatted in various styles (single column, multiple column, report, book, et.), font types (Roman, Helvetica, etc.), and font size (9, 10, and 12 pt., etc.). Thus, the performance of any character recognition system can be studied by providing as input the same (or different) text formatted in various styles with varied but controlled degradation.

X. CONCLUSION

We described a model for the perspective distortion occurring during the photocopying and scanning process. This model accounts for the physical deformation of the document page, perspective distortion, nonlinear intensity variations, and nonlinear optical point-spread function. We also described a morphological model for local distortions in terms of distance transforms and morphological closing operation. Simulation results were given for both models. Two issues that have not been addressed here are degradation model parameter estimation and model validation; these issues have been addressed in Kanungo et al. [15].

ACKNOWLEDGMENTS

(a)
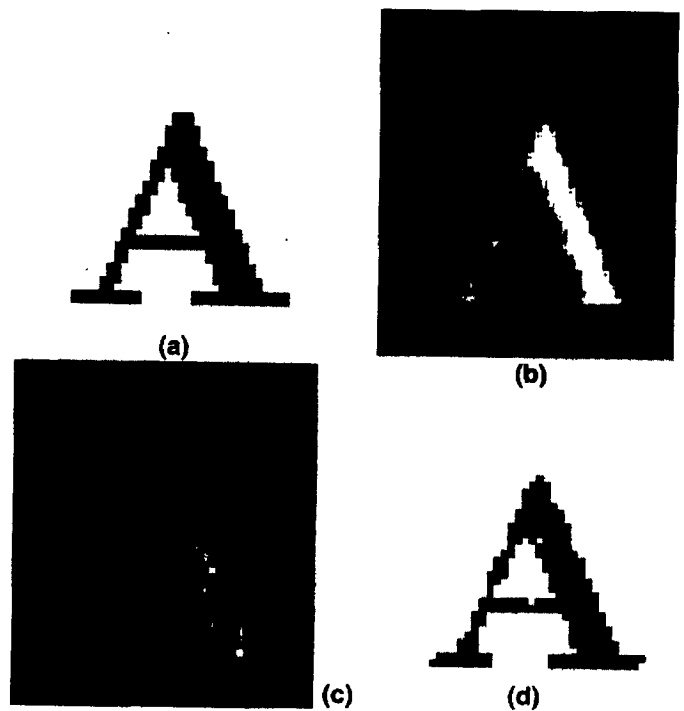

(b)

(c)

(d)

Figure 9. Baird's document defect model: (a) ground truth data; (b) ground truth image (a) convolved with a 5×5 Gaussian with $\sigma = 0.7$; (c) the resultant image (b) with zero mean a $\sigma = 16$ Gaussian noise; (d) the resultant image thresholded at $T = 75$.

IX. IMPLEMENTATION

The noise-free documents were created from Latex formatted documents. The LaTex-formatted ASCII file was converted into a device-independent format (DVI) using Latex. Next, a public-domain dvi file previewer, XDVI, was modified to produce one-bit/pixel images in TIFF format. The perspective distortion model has been implemented and tested in the GIPSY Image Processing Package. The local document degradation model has been implemented in C language, and is available on a CD-ROM (please contact Dr. Haralick for more information).

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ERB, Woody, Pheff, Bont, Tranman, IP, Dalton, Christine and OOZ

Abstract—The theoretical analysis and derivation of artificial neural systems consist essentially of manipulating symbolic mathematical objects according to certain mathematical and biological knowledge. A simple observation has been made that this work can be done more efficiently with computer assistance by using and extending methods and systems of symbolic computation. In this paper, after presenting the mathematical characteristics of neural systems and a brief review on Liapunov stability theory, we present some features and capabilities of existing systems and our extension for manipulating objects occurring in the analysis of neural systems. Then, some strategies and a toolkit developed in MACSYMA for computer aided analysis and derivation are described. A concrete example is given to demonstrate the derivation of a hybrid neural system, i.e. a system which in its learning rule combines elements of supervised and unsupervised learning. The future work and directions on this topic are indicated.

Keywords— CA system, computer aided analysis and derivation, Liapunov function, neural system, symbolic computation.

I. INTRODUCTION

SINCE the early 1940s a large number of artificial neural systems have been proposed by neural scientists. The dynamical behavior of these systems may be mathematically described by sets of coupled equations like differential equations for formal neurons with graded response. The investigation of essential features of neural systems such as stability and adaptation depends strongly upon the state of the mathematical theory to be applied and on a concrete and efficient analysis of dynamical equations. Unlike abstract theoretical research in which the mathematical objects adopted are frequently assumed to be of certain canonical form, the neurodynamics is usually complicated due to various biological facts which should be taken account of to a degree as large as possible. Consequently, this makes the analysis and derivation very complex, sometimes to an extent which is beyond human capacity, and the traditional methods and tools of mathematics are not always sufficient. It is therefore proposed in [19] to use and extend the methods and software systems of symbolic computation for handling, analyzing and constructing neurodynamics and its related objects. The present paper is the continuation of our work in this direction. The attempt is to demonstrate how symbolic computation can be applied to aid the analysis and derivation of neural systems.

In contrast to the approximative character of numerical calculations, symbolic computation treats objects with semantics like functions, formulae and programs. A variety of software systems for performing symbolic computation have been developed for research and applications in natural and technical sciences. However, the existing systems cannot be directly used for the analysis and derivation of neural systems as the operations on the occurring objects, particularly those involving an unspecified number of arguments like indefinite summations, have not yet been taken into account. To achieve our goal, some rules for differentiating and integrating indefinite summations with respect to indexed variables were proposed [20]. A toolkit has been designed and implemented in MACSYMA for manipulating these objects occurring in the analysis and derivation of neural systems [21].

In the next section, we introduce the general method and techniques for the stability analysis of artificial neural systems. The role of symbolic computation for representing and manipulating the objects concerning neural systems is discussed in Section III. In Section IV we present some strategies for using computer algebra (CA) systems and their extension to analyse the stability of neural systems and to derive novel stable systems. A brief description of a toolkit developed in MACSYMA is also provided. A concrete example is given in Section V to illustrate the derivation of a hybrid model by our toolkit. Section VI contains a discussion on future developments. The paper is closed with a brief summary.

II. STABILITY ANALYSIS OF NEURAL SYSTEMS

Consider artificial neural systems which are described by coupled systems of differential equations of the form

$$\dot{x} = F(x, w, K) \tag{1}$$

and

$$\dot{w} = G(x, w, K) \tag{2}$$

where $x = (x_1(t), ..., x_n(t))$ is the activation state vector, $w = (w_{ij}(t))$ is the weight matrix of dimension $n \times n$, n is the number of nodes and K is an external time-independent pattern vector. Such systems of differential equations which describe the neural model will occasionally be named *neurodynamics*.

Once a neural model is proposed, its main features are represented by its dynamic behavior. The adaptability of

Figure 10. Noise-free document. The size of the image is 3300 × 2550.

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Figure 11. The resultant perturbed image with model parameters $\alpha_0 = 1.0$, $\alpha = 0.4$, $\beta = \beta_0 = \eta = 0$. The closing was performed with a 4 × 4 structuring element.

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where $x = (x_1(t), ..., x_r(t))$ is the activation state vector, $w = (w_{ij}(t))$ is the weight matrix of dimension $n \times n$, n is the number of nodes and K is an external time-independent pattern vector. Such systems of differential equations which describe the neural model will occasionally be named *neurodynamics*.

Once a neural model is proposed, its main features are represented by its dynamic behavior. The adaptability of

Figure 12. The resultant perturbed image with model parameters $\alpha_0 = 1.0$, $\alpha = 0.2$, $\beta = \beta_0 = \eta = 0.0$. The closing was performed with a 4×4 structuring element.

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ERB, Woody, Pheff, Bont, Tranman, IP, Dalton, Christine and OOZ

Abstract—The theoretical analysis and derivation of artificial neural systems consist essentially of manipulating symbolic mathematical objects according to certain mathematical and biological knowledge. A simple observation has been made that this work can be done more efficiently with computer assistance by using and extending methods and systems of symbolic computation. In this paper, after presenting the mathematical characteristics of neural systems and a brief review on Liapunov stability theory, we present some features and capabilities of existing systems and our extension for manipulating objects occurring in the analysis of neural systems. Then, some strategies and a toolkit developed in MACSYMA for computer aided analysis and derivation are described. A concrete example is given to demonstrate the derivation of a hybrid neural system, i.e. a system which in its learning rule combines elements of supervised and unsupervised learning. The future work and directions on this topic are indicated.

Keywords—CA system, computer aided analysis and derivation, Liapunov function, neural system, symbolic computation.

I. INTRODUCTION

SINCE the early 1940s a large number of artificial neural systems have been proposed by neural scientists. The dynamical behavior of these systems may be mathematically described by sets of coupled equations like differential equations for formal neurons with graded response. The investigation of essential features of neural systems such as stability and adaptation depends strongly upon the state of the mathematical theory to be applied and on a concrete and efficient analysis of dynamical equations. Unlike abstract theoretical research in which the mathematical objects adopted are frequently assumed to be of certain canonical form, the neurodynamics is usually complicated due to various biological facts which should be taken account of to a degree as large as possible. Consequently, this makes the analysis and derivation very complex, sometimes to an extent which is beyond human capacity, and the traditional methods and tools of mathematics are not always sufficient. It is therefore proposed in [19] to use and extend the methods and software systems of symbolic computation for handling, analysing and constructing neurodynamics and its related objects. The present paper is the continuation of our work in this direction. The attempt is to demonstrate how symbolic computation can be applied to aid the analysis and derivation of neural systems.

In contrast to the approximative character of numerical calculations, symbolic computation treats objects with semantics like functions, formulae and programs. A variety of software systems for performing symbolic computation have been developed for research and applications in natural and technical sciences. However, the existing systems cannot be directly used for the analysis and derivation of neural systems as the operations on the occurring objects, particularly those involving an unspecified number of arguments like indefinite summations, have not yet been taken into account. To achieve our goal, some rules for differentiating and integrating indefinite summations with respect to indexed variables were proposed [20]. A toolkit has been designed and implemented in MACSYMA for manipulating these objects occurring in the analysis and derivation of neural systems [21].

In the next section, we introduce the general method and techniques for the stability analysis of artificial neural systems. The role of symbolic computation for representing and manipulating the objects concerning neural systems is discussed in Section III. In Section IV we present some strategies for using computer algebra (CA) systems and their extension to analyse the stability of neural systems and to derive novel stable systems. A brief description of a toolkit developed in MACSYMA is also provided. A concrete example is given in Section V to illustrate the derivation of a hybrid model by our toolkit. Section VI contains a discussion on future developments. The paper is closed with a brief summary.

II. STABILITY ANALYSIS OF NEURAL SYSTEMS

Consider artificial neural systems which are described by coupled systems of differential equations of the form

$$\dot{s} = F(s, w, K) \tag{1}$$

and

$$\dot{w} = G(s, w, K) \tag{2}$$

where $s = (s_1(t), ..., s_n(t))$ is the activation state vector, $w = (w_{ij}(t))$ is the weight matrix of dimension $n \times n$, n is the number of nodes and K is an external time-independent pattern vector. Such systems of differential equations which describe the neural model will occasionally be named *neurodynamics*.

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Figure 13. The resultant perturbed image with model parameters $\alpha_0 = \alpha = 0.0$, $\beta_0 = 1.0$, $\beta = 0.4$, $\eta = 0.0$. The closing was peformed with a 4×4 structuring element.

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Figure 14. The resultant perturbed image with model parameters $\alpha_0 = \alpha = 0.0$, $\beta_0 = 1.0$, $\beta = 0.3$, $\eta = 0.0$. The closing was performed with a 4×4 structuring element.

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